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CONDUCTION OF HEAT THROUGH THE THIN AND STRAIGHT TRIANGULAR FIN

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ABSTRACT

Heat conduction takes place through the medium from particle to particle without the actual movements of the particles of the medium due to temperature gradient in the direction where temperature decreases. In practical situations where maximum heat conduction per unit weight at low cost is required the fins (also known as extended surfaces) of varying cross-sections like triangular fins or hyperbolic fins or parabolic fins are used. The paper discusses the theory of thin and straight triangular fin to obtain the heat conducted through it via means of the Elzaki Transform Method.

Index terms: Thin and straight triangular fin, Elzaki Transform method, Heat Conducted.

INTRODUCTION

Fins are the extended surfaces which are mostly used in the devices which exchange heat like central processing unit, power plants, radiators heat sinks, etc. It has been noted that the heat is not dissipated uniformly by each element of the fin but is dissipated mostly near the base of the fin. Thus if we use a uniform fin, then there would be wastage of material. Because of these aspects, straight triangular fin was developed [1], [2]. The paper obtains the heat conducted through the thin and straight triangular fin via means of Elzaki Transform Method. It is usually obtained by ordinary Calculus method [1], [3]. Here Elzaki Transform Method comes out to be very effective tool to obtain the heat conducted through the thin and straight triangular fin.

1. BASIC DEFINITIONS

2.1 Elzaki Transform

The Elzaki transform [4], [5] of $\hbar(y), y \geq 0$ is defined as

$$\mathrm{E}\{\hbar(y)\} = \mathcal{H}(p) = p\int_0^\infty e^{-\frac{y}{p}}\hbar(y)dy.$$

The Elzaki Transform of some elementary functions [5], [6] are given as

- $E\{y^n\} = n!\,p^{n+2}, where\ n = 0,1,2, \ldots\ldots$
- $E\{e^{ay}\} = \frac{p^2}{1-ap},$
- $E\{sinay\} = \frac{ap^3}{1+a^2p^2},$
- $E\{cosay\} = \frac{ap^2}{1+a^2p^2},$

2.2 Elzaki Transform of Derivatives

The Elzaki Transform of derivatives [5], [6] of $\mathrm{h}(y)$ are given by

- $E\{h'(y)\} = \frac{1}{p}E\{h(y)\} - p\,h(0)$

  $or\ E\{\hbar'(y)\} = \frac{1}{p}\mathcal{H}(p) - p\,\hbar(0),$
- $E\{\hbar''(y)\} = \frac{1}{p^2}\mathcal{H}(p) - \hbar(0) - p\hbar'(0),$

  $E\{y\hbar'(y)\} = p^2\frac{d}{dp}\left\{\frac{1}{p}\mathcal{H}(p) - p\,\hbar(0)\right\} - p\left\{\frac{1}{p}\mathcal{H}(p) - p\,\hbar(0)\right\},$
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- $E\{y\, \hbar''(y)\} = p^2 \frac{d}{dp}\left\{\frac{1}{p^2}\mathcal{H}(p) - \hbar(0) - p\hbar'(0)\right\} - p\left\{\frac{1}{p^2}\,\mathcal{H}(p) - \hbar(0) - p\hbar'(0)\right\}$

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2. MATERIAL AND METHOD

The differential equation for analyzing a thin and straight triangular fin [1] (so that heat flow pertains to one-dimensional heat conduction) is given by

$$\theta''(x) + \frac{1}{x}\theta'(x) - \frac{D^2}{x}\theta(x) = 0 \ldots\ldots (1),$$

$where\ D = \sqrt{\frac{2hL}{tk}}$, L is the length of the fin between the base at x = L and the tip at x = 0, t is the thickness of the fin which increases uniformly from zero at the tip to t at the base, k is thermal conductivity, h is the coefficient of transfer of heat by convection, $\theta(x) = T(x) - T_s$, T_s is the temperature of the environment and T_0 is the temperature at the base at x = 0 of the fin.

Multiplying both sides of (1) by x, we get

$$x\,\theta''(x) + \theta'(x) - D^2\theta(x) = 0\ldots\ldots (2)$$

The Elzaki transform of (2) gives

$$p^2\frac{d}{dp}\left[\frac{\theta(p)}{p^2} - \theta(0) - p\theta'(0)\right] - p\left[\frac{\theta(p)}{p^2} - \theta(0) - p\theta'(0)\right] + \frac{\theta(p)}{p} - p\theta(0) - D^2\theta(p) = 0$$

Put $\theta(0) = b\ and\ \theta'(0) = a$, and simplifying and rearranging the equation, we get

$$\frac{\theta'(p)}{\theta(p)} = \frac{2}{p} + D^2\ldots\ldots (4)$$

Integrating both sides of equation w.r.t. p and simplifying, we get

$$\log_e \theta(p) = 2\log_e p + D^2 p + \log_e c$$

Simplifying the equation, we get

$$\theta(p) = c\,p^2 e^{(D^2 p)}$$

Or

$$\theta(p) = c\,p^2\left[1 + D^2 p + \frac{(D^2p)^2}{2!} + \frac{(D^2p)^3}{3!} + \frac{(D^2p)^4}{4!}\ldots\ldots\right]$$

$$or\ \theta(p) = c\left[p^2 + D^2p^3 + \frac{D^4p^4}{2!} + \frac{D^6p^5}{3!} + \frac{D^8p^6}{4!}\ldots\ldots\right]\ldots\ldots..(4)$$

The inverse Elzaki transform of (4) provides

$$\theta(x) = c\left[1 + D^2x + \frac{D^4x^2}{2!\,2!} + \frac{D^6x^3}{3!\,3!} + \frac{D^8x^4}{4!\,4!}\ldots\right]$$

$$or\ \theta(x) = c\left[\begin{array}{l} 1 + \frac{1}{4}\left(2D\sqrt{x}\right)^2 + \frac{1}{2!\,2!}\left(\frac{2D\sqrt{x}}{2}\right)^4 \\ + \frac{1}{3!\,3!}\left(\frac{2D\sqrt{x}}{2}\right)^6 + \frac{1}{4!\,4!}\left(\frac{2D\sqrt{x}}{2}\right)^8\ldots \end{array}\right]\ldots..(5)$$

The first kind modified Bessel function [7] of order n is given by

$$I_n(z) = \sum_{r=0}^{\infty}\frac{1}{r!(n+r)!}\left(\frac{z}{2}\right)^{n+2r}\ldots\ldots. (6)$$

Also $\frac{d}{dx}\left(I_n(z)\right) = I_{n+1}(z)\frac{d}{dx}(z)\ldots. (7)$

Put z = $2D\sqrt{x}$ and n = 0, we get

$$I_0\left(2D\sqrt{x}\right) = \sum_{r=0}^{\infty}\frac{1}{r!\,r!}\left(\frac{2D\sqrt{x}}{2}\right)^{2r}$$

Or

$$I_0\left(2D\sqrt{x}\right) = 1 + \left(\frac{2D\sqrt{x}}{2}\right)^2 + \frac{1}{2!2!}\left(\frac{2D\sqrt{x}}{2}\right)^4 + \frac{1}{3!3!}\left(\frac{2D\sqrt{x}}{2}\right)^6 + \frac{1}{4!4!}\left(\frac{2D\sqrt{x}}{2}\right)^8\ldots\ldots$$

Hence (5) becomes

$$\theta(x) = c I_0\left(2D\sqrt{x}\right)\ldots. (8)$$

To find c, at x = L, $\Theta(L) = \Theta_0$, therefore, c = $\frac{\Theta_0}{I_0(2D\sqrt{L})}$

Hence (8) can be written as

$$\Theta(x) = \frac{\Theta_0}{I_0(2D\sqrt{L})} I_0(2D\sqrt{x})\ldots (9)$$

The equation (9) gives the temperature distribution along the length of the thin and straight triangular fin.

According to the Fourier's law of conduction of heat, the heat conducted [8], [9], [10], [11] through the fin (thin and straight triangular) is given by the equation

$$H = kA\left(\Theta'(x)\right)_{x=L} = kbt\left(\Theta'(x)\right)_{x=L}$$

Using equation (9), we get

$$H = kbt\,\frac{\Theta_0}{I_0(2D\sqrt{L})} I_1\left(2D\sqrt{L}\right)\left(\frac{d}{dx}\left(2D\sqrt{x}\right)\right)_{x=L}$$

On simplifying the equation, we get

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$$H = kbtD \frac{\theta_0}{\sqrt{L} I_0(2D\sqrt{L})} I_1(2D\sqrt{L}) \ldots. (10)$$

Put the value of D, we get

$$H = b\sqrt{2hkt} \frac{\theta_0}{I_0(2D\sqrt{L})} I_1(2D\sqrt{L}) \ldots \ldots (11)$$

This equation (11) gives the expression for the rate of heat flow through the thin and straight triangular fin.

3. CONCLUSION AND FUTURE SCOPE

In this paper, the heat conducted through the thin and straight triangular fin has been successfully derived via means of the Elzaki Transform Method. It is observed that with the increase in the length of the thin and straight triangular fin, temperature increases and hence conduction of heat at any cross-section of the thin and straight fin is more. The method has come out to be a very effective tool to obtain the heat conducted through the thin and straight triangular. The method can also be used in solving problems arising in the disciplines: network analysis, heat and mass transfer, quantum mechanics, classical mechanics, etc.

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