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APPLICATION OF NOVEL INTEGRAL TRANSFORM: GUPTA TRANSFORM TO MECHANICAL AND ELECTRICAL OSCILLATORS

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ABSTRACT

In this paper, the responses of mechanical and electrical oscillators are obtained by the application of a novel integral transform called Gupta Transform. This paper proposed the Gupta Transform as a novel approach for analyzing the mechanical and electrical oscillators. Like other integral Transforms or methods or approaches, the Gupta Transform would also present a simple and effective mathematical tool for obtaining the responses of mechanical and electrical oscillators.

Keywords: Mechanical and Electrical oscillators, Response, Gupta Transform.

I. INTRODUCTION

This paper discusses the theory of mechanical and electrical oscillators to obtain their responses by the application of new integral transform called Gupta Transform. It was propounded recently by the authors Rohit Gupta and Rahul Gupta. The authors applied the Gupta Transform to the initial value problems in Science and Engineering [1]. Generally, the Mechanical and electrical oscillators have been analyzed by the calculus method [2], [3] or matrix method [4] or Laplace Transform [5]. This paper proposed the Gupta Transform as a novel approach for finding out the responses of mechanical and electrical oscillators.

I. GUPTA TRANSFORM

Let g(y) is a continuous function on any interval for y ≥ 0. The Gupta Transform of g(y) is defined as [1]

$\dot{R}\{g(y)\} = \frac{1}{q^3}\int_0^\infty e^{-qy}\, g(y)dy = G(q),$ provided that the integral is convergent, where q may be a real or complex parameter and $\dot{R}$ is the Gupta Transform operator.

The Gupta Transform of some elementary functions [1] are

- $\dot{R}\{y^n\} = \frac{n!}{q^{n+4}}$, $where\ n = 0,1,2,3 \ldots \ldots \ldots$
- $\dot{R}\{e^{by}\} = \frac{1}{q^3(q-b)}$, $r > b$
- $\dot{R}\{sinby\} = \frac{b}{q^3(q^2+b^2)}$, $r > 0$
- $\dot{R}\{sinhby\} = \frac{b}{q^2(q^2-b^2)}$, $r > |b|$
- $\dot{R}\{cosby\} = \frac{1}{q^2(q^2+b^2)}$, $r > 0$
- $\dot{R}\{coshby\} = \frac{1}{q^2(q^2-b^2)}$, $r > |b|$
- $\dot{R}\{\delta(y-b)\} = \frac{1}{q^4}\, e^{-bq}$

Inverse Gupta Transform of Elementary Functions

The inverse Gupta Transform of the function [1] G(r) is denoted by $\dot{R}^{-1}$\{G (r)\} or g (y).

If we write $\dot{R}$ \{g (y)\} = G (r), then $\dot{R}^{-1}$\{G (r)\} = g (y), where $\dot{R}^{-1}$ is called the inverse Gupta Transform operator.

The Inverse Gupta Transform of some elementary functions [1] are given below

- $\dot{R}^{-1}\{1/q^n\} = \frac{y^{n-4}}{(n-4)!}$
- $\dot{R}^{-1}\left\{\frac{1}{q^3(q-b)}\right\} = e^{by}$
- $\dot{R}^{-1}\left\{\frac{1}{q^3(q^2+b^2)}\right\} = \frac{1}{b}\, sinby$

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- $\dot{R}^{-1}\left\{\frac{1}{q^2(q^2-b^2)}\right\} = \frac{1}{b} sinhby$
- $\dot{R}^{-1}\left\{\frac{1}{q^2(q^2+b^2)}\right\} = cosby$
- $\dot{R}^{-1}\left\{\frac{1}{q^2(q^2-b^2)}\right\} = coshby$

Gupta Transform of derivatives

Let g(y) is continuous function and is piecewise continuous on any interval, then the Gupta Transform of first derivative [1] of g(y) i.e. $\dot{R}\{g'(y)\}$ is given by

$$\dot{R}\{g'(y)\} = \frac{1}{q^3}\int_0^\infty e^{-qy}\, g'(y)dy$$

Integrating by parts and applying limits, we get

$\dot{R}\{g'(y)\}$

$= \frac{1}{q^3}\{-g(0) - \int_0^\infty -qe^{-qy}\, g(y)dy\} = \frac{1}{q^3}\{-g(0) + q\int_0^\infty e^{-qy}\, g(y)dy\}$

$$= qG(q) - \frac{1}{q^3}g(0)$$

Hence

$$\dot{R}\{g'(y)\} = qG(q) - \frac{1}{q^3}g(0)$$

Since

$\dot{R}\{g'(y)\} = q\dot{R}\{g(y)\} - \frac{1}{q^3}g(0)$, Therefore, on replacing g(y) by $g'(y)$ and $g'(y)$ by $g''(y)$, we have

$$\dot{R}\{g''(y)\} = q\dot{R}\{g'(y)\} - \frac{1}{q^3}g'(0)$$

$$= q\left\{q\dot{R}\{g(y)\} - \frac{1}{q^3}g(0)\right\} - \frac{1}{q^3}g'(0)$$

$$= q^2\dot{R}\{g(y)\} - \frac{1}{q^2}g(0) - \frac{1}{q^3}g'(0)$$

$$= q^2G(q) - \frac{1}{q^2}g(0) - \frac{1}{q^3}g'(0)$$

Hence

$$\dot{R}\{g''(y)\} = q^2G(q) - \frac{1}{q^2}g(0) - \frac{1}{q^3}g'(0)$$

And so on.

II. MATERIAL AND METHOD

SIMPLE HARMONIC OSCILLATOR

The differential equation of simple harmonic oscillator [2, 5] is given by

$\ddot{x}(t) + \omega^2 x(t) = 0$ .... (1),

$\omega = \sqrt{\frac{K}{m}}$ is the natural frequency of the oscillator.

We assume:

(i) At t = 0, x (0) = 0.

(ii) Also, at t = 0, the velocity of the simple harmonic oscillator i.e. $\dot{x}(0) = v_0$.

The Gupta Transform of (1) provides

$q^2G(q) - \frac{1}{q^2}x(0) - \frac{1}{q^3}x'(0) + \omega^2 G(q) = 0$... (2)

Here G (q) denotes the Gupta Transform of $x(t)$.

Put $x(0) = 0$ and $\dot{x}(0) = v_0$ and simplifying (2), we get

$G(q) = \frac{v_0}{q^3(q^2+\omega^2)}$..... (3)

Applying inverse Gupta Transform, we get

$x(t) = \frac{v_0}{\omega} sin\omega y$ .. (4)

This equation shows that the motion of a simple harmonic oscillator is oscillatory with constant amplitude.

DAMPED MECHANICAL OSCILLATOR

The differential equation of damped Mechanical oscillator [4], [5] is given by

$\ddot{x}(t) + 2a\dot{x}(t) + \omega^2 x(t) = 0$ .... (5),

$2a = \frac{r}{m}$ is the damping constant per unit mass, $\omega = \sqrt{\frac{K}{m}}$ is the natural frequency of the oscillator. For a light damping, $a < \omega$.

We assume [4], [5]:

i. At t = 0, x (0) = 0.

ii. Also, at t = 0, the velocity of the damped mechanical oscillator i.e. $\dot{x}(0+) = v_0$.

The Gupta transform of (1) provides

$q^2G(q) - \frac{1}{q^2}x(0) - \frac{1}{q^3}x'(0) + 2a\left\{qG(q) - \frac{1}{q^3}x(0)\right\} + \omega^2 G(q)$

= 0... (6)

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Here G (q) denotes the Gupta transform of $x(t)$.
Put $x(0) = 0$ and $\dot{x}(0) = v_0$ and simplifying (6), we get

$$G(q) = \frac{v_0}{q^3(q^2 + 2a\,q + \omega^2)}$$

Or

$$G(q) = \frac{v_0}{q^3(q+\beta_1)\,(q+\beta_2)}$$

$where\ \beta_1 = a + i\sqrt{\omega^2 - a^2}\ ,\beta_2 = a - i\sqrt{\omega^2 - a^2}\ ,\ \beta_1 - \beta_2 = 2i\sqrt{\omega^2 - a^2}$

.

Or

$$G(q) = \frac{v_0}{(\beta_2-\beta_1)q^3(q+\beta_1)} - \frac{v_0}{(\beta_2-\beta_1)q^3(q+\beta_2)}$$

Applying inverse Gupta Transform, we get

$$x(t) = v_0 \frac{\exp(-\beta_1 t) - \exp(-\beta_2 t)]}{(\beta_2-\beta_1)}$$

Or

$$x(t) = v_0 e^{-at} \frac{[\exp(i\sqrt{\omega^2-a^2}t) - \exp(-i\sqrt{\omega^2-a^2}t)]}{2i\sqrt{\omega^2-a^2}}$$

Or

$$x(t) = \frac{v_0 e^{-at}}{\sqrt{\omega^2-a^2}} \sin\sqrt{\omega^2 - a^2}\,t \text{.... (7)}$$

This equation shows that the motion of a lightly damped oscillator is oscillatory with decreasing amplitude.

For an **overdamped oscillator** [6], $a > \omega$, therefore, replacing $\sqrt{\omega^2 - a^2}$ by $i\sqrt{a^2 - \omega^2}$ in (7), the displacement of an overdamped oscillator is given by

$$x(t) = \frac{v_0 e^{-at}}{i\sqrt{a^2-\omega^2}} \sin i\sqrt{a^2 - \omega^2}\,t$$

Or

$$x(t) = \frac{v_0 e^{-at}}{\sqrt{a^2-\omega^2}} \sinh\sqrt{a^2 - \omega^2}\,t \text{.... (8)}$$

This equation shows that the motion of a heavily damped oscillator is non-oscillatory.

DAMPED ELECTRICAL OSCILLATOR

The differential equation of electrical oscillator (LRC circuit) [8], [9] is given by

$Q(t) + 2a\dot{Q}(t) + \omega^2 Q(t) = 0$ .... (9), where

$\omega = \sqrt{\frac{1}{LC}}$ is the angular frequency of the electrical oscillator, $2a = \frac{R}{L}$ is the damping coefficient. $Q(t)$ is the charge at any instant.

We assume [10]:

(i) At t = 0, Q (0) = 0
(ii) Also, at t = 0, the current in the circuit i.e. $\dot{Q}(0^+) = i_0$

The Gupta Transform of (9) provides

$$q^2G(q) - \frac{1}{q^2}x(0) - \frac{1}{q^3}x'(0) + 2a\left\{qG(q) - \frac{1}{q^3}x(0)\right\} + \omega^2 G(q)$$

= 0...(10)

Here G (q) denotes the Gupta transform of $Q(t)$.

Put $Q(0) = 0$ and $\dot{Q}(0) = i_0$ and simplifying (10), we get

$$G(q) = \frac{i_0}{q^3(q^2 + 2a\,q + \omega^2)}$$

Or

$$G(q) = \frac{i_0}{q^3(q+\beta_1)\,(q+\beta_2)}$$

$where\ \beta_1 = a + i\sqrt{\omega^2 - a^2}\ ,\beta_2 = a - i\sqrt{\omega^2 - a^2}\ ,\ \beta_1 - \beta_2 = 2i\sqrt{\omega^2 - a^2}$

Or

$$G(q) = \frac{i_0}{(\beta_2-\beta_1)q^3(q+\beta_1)} - \frac{i_0}{(\beta_2-\beta_1)q^3(q+\beta_2)}$$

Applying inverse Gupta Transform, we get

$$Q(t) = i_0 \frac{\exp(-\beta_1 t) - \exp(-\beta_2 t)]}{(\beta_2-\beta_1)}$$

Or

$$Q(t) = i_0 e^{-at} \frac{[\exp(i\sqrt{\omega^2-a^2}t) - \exp(-i\sqrt{\omega^2-a^2}t)]}{2i\sqrt{\omega^2-a^2}}$$

Or

$$Q(t) = \frac{i_0 e^{-at}}{\sqrt{\omega^2-a^2}} \sin\sqrt{\omega^2 - a^2}\,t \text{.... (11)}$$

This equation shows that the behaviour of oscillator (charge) is oscillatory with the amplitude of oscillations decreases with time exponentially. The decrease in amplitude i.e. damping depends upon resistance R in the circuit. Such damping is called resistance damping [8]. If R = 0, the amplitude would remain constant. Hence in the LRC circuit, the resistance is the only dissipative element.

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III. CONCLUSION

In this paper, the responses of mechanical and electrical oscillators has been successfully obtained by the application of a new integral transform called Gupta Transform and proposed the Gupta Transform for discussing the theory of a mechanical and electrical oscillators. A novel and different method have been exploited for obtaining the responses of mechanical and electrical oscillators.

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